Sample Questions 2-I

Wednesday, May 1, 2019 7:40 AM

(1) which of the following is not a subspace of $\mathbb{R}^2$.

(a) $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x+y \leq 1 \right\}$ (c) $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} x-y \\ x+y \end{bmatrix} : x, y \in \mathbb{Z} \right\}$ (d) $\left\{ t \begin{bmatrix} 1 \\ 1 \end{bmatrix} : t \in \mathbb{R} \right\}$

(2) which of the following is not a subspace of $C[a,b]$, the vector space of continuous functions on the interval $[a,b]$.

(a) $\mathbb{P}_3$ (b) $\{ f \in C[a,b] : f(a) = f(b) \}$.

(c) $\{ f \in C[a,b] : f(a) - f(b) = -1 \}$

(d) $\{ f \in C[a,b] : f(a) = -f(a) \}$.

(3) which of the following is not a subspace of $\mathbb{P}_n$.

(a) All polynomials p in $\mathbb{P}_n$ such that $p(0) = 0$.

(b) $\{ at : a \in \mathbb{R} \}$

(c) $\{ a + t^2 : a \in \mathbb{R} \}$

(d) $\mathbb{P}_n$

(4) Mark each statement True or False.

(a) If f is a function in the vector space V of all real-valued functions on $\mathbb{R}$ and if $f(t) = 0$ for some t, then f is the zero vector in V.

(b) A subspace also is a vector space.

(c) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

(d) A subset H of a vector space V is a subspace of V if the following conditions are satisfied: (i) the zero vector of V is in H, (ii) for all $u, v \in V$, $u + v \in H$, (iii) for all $c \in \mathbb{R}$ and $u \in H$, $cu \in H$.

(e) Let V be a vector space and $v_1, v_2 \in V$. Then $\text{span}\{v_1, v_2\}$ is a subspace of V.

Sample Questions 2-II

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(5) which of the following vectors is not in Nul A where

$$A = \begin{bmatrix} 1 & -6 & 4 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

(a) $\begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 3/2 \\ 1 \\ 0 \end{bmatrix}$

(6) Let A be an $m \times n$ matrix. Mark each statement True or False.

(a) The null space of A is the solution set of $Ax = 0$.

(b) The column space of A is the rang of the mapping $x \mapsto Ax$.

(c) A null space is a vector space.

(d) The column space of A is in $\mathbb{R}^m$.

(c) Fix $b \in \mathbb{R}^m$. Then $\text{Col}\, A = \{x : Ax = b\}$.

(d) The range of a linear transformation is a vector space.

(7) which of the following is not a linear transformation.

(a) $T: \mathbb{P}_2 \longrightarrow \mathbb{R}^2$
$p \longmapsto \begin{bmatrix} p(0) \\ p(0) \end{bmatrix}$

(b) $T: M_{2\times 2}(\mathbb{R}) \longrightarrow M_{2\times 2}(\mathbb{R})$
$A \longmapsto A + A^T$

(c) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$
$x \longmapsto Ax$
where $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$

(d) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}$
$A \longmapsto \det(A)$

(8) Mark each statement True or False.

(a) A single vector by itself is linearly independent.

(b) If $H = \text{Span}\{b_1, \dots, b_p\}$, then $\{b_1, \dots, b_p\}$ is a basis for H.

(c) The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.

(d) If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V.

Sample questions 2-IV

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(13) The vector space $\mathbb{P}_3$ of the polynomials of degree at most 3 ...

(a) is isomorphic to $\mathbb{R}^3$

(b) has dimension 3

(c) is isomorphic to any vector space of dimension 4.

(d) has only 4 elements.

(14) Let $T: V \to W$ be an isomorphism of vector spaces.

(a) $\dim V = \dim W$

(b) It is possible that $\{v_1, \ldots, v_n\}$ is linearly independent and $\{T(v_1), \ldots, T(v_n)\}$ is linearly dependent.

(c) It is possible that $v_1 \in \text{span}\{v_2, \ldots, v_n\}$ but $T(v_1) \notin \text{Span}\{v_2, \ldots, v_n\}$

(d) None of the above are true.

(15) Let V be a vector space and V has a basis $\{b_1, \ldots, b_n\}$. Then

(a) any basis of V must have exactly n vectors.

(b) If $v \in V$, then $\{b_1, \ldots, b_n, v\}$ is linearly dependent.

(c) If $v \in V$, then $\{b_1, \ldots, b_n, v\}$ is linearly independent.

(d) It is possible that we can not span V by finitely many vectors.

(16) If V is a p-dimensional vector space. Then which of the following is not true.

(a) Any linearly independent set of exactly p elements of V spans V.

(b) Any linearly independent set of exactly p elements of V is a basis for V.

(c) Any set of exactly p elements of V is linearly independent.

(d) All of the above are not true.

Sample Questions 2-V

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(17) Let $\mathcal{B}$ and C be bases for V. Then

(a) there is a unique matrix $\underset{C\leftarrow B}{P}$ such that $[x]_C = \underset{C\leftarrow B}{P}[x]_B$.

(b) $\underset{C\leftarrow B}{P} = \left[\,[b_1]_C \;[b_2]_C \cdots [b_n]_C\,\right]$ where $\mathcal{B}=\{b_1,\dots,b_n\}$.

(c) $\underset{C\leftarrow B}{P} = \left[\,[c_1]_{\mathcal{B}} \;[c_2]_{\mathcal{B}} \cdots [c_n]_{\mathcal{B}}\,\right]$ where $C=\{c_1,\dots,c_n\}$.

(d) $\underset{C\leftarrow B}{P}^{-1} = \left[\,[c_1]_{\mathcal{B}} \;[c_2]_{\mathcal{B}} \cdots [c_n]_{\mathcal{B}}\,\right]$ where $C=\{c_1,\dots,c_n\}$.

(18) Let $\mathcal{B}$ and C be bases for a vector space V. Then

(a) $[x]_C = P_C^{-1} P_{\mathcal{B}} [x]_{\mathcal{B}}$.

(b) $[x]_{\mathcal{B}} = P_C^{-1} P_{\mathcal{B}} [x]_C$

(c) $[x]_C = \underset{C\leftarrow B}{P} [x]_{\mathcal{B}}$.

(d) $P_C^{-1} P_{\mathcal{B}} = \underset{C\leftarrow B}{P}$.

(19) which one is not true.

(a) zero vector cannot be an eigenvector.

(b) The set of eigenvalues of a matrix A is the roots of $\det(A-\lambda I)$.

(c) If λ_1 and λ_2 are the only roots of $\det(A-\lambda I)$, where A an $n\times n$ matrix, then the sum of the dimensions of eigenspaces corresponding to λ_1 and λ_2 is equal to n.

(d) zero can be an eigenvalue.

Sample Questions 2-VI

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(20) when 0 is an eigenvalue of A, then this is not true that

(a) A is invertible.

(b) The eigenspace corresponding to 0 is the same as the Nul A.

(c) 0 is a root of $\det(A-\lambda I)$.

(d) any of the above is not true.

(21) If A is an $n \times n$ matrix. Then A is diagonalizable if and only if

(a) $\det(A-\lambda I)$ has n distinct roots.

(b) A has n distinct eigenvectors.

(c) At least there is an eigenvalue λ such that the dimension of its eigenspace is greater than 1.

(d) For every diagonal matrix D, there is an invertible matrix P such that $D = P^{-1}AP$.

(22) Let A be an $n \times n$ matrix that the set of its columns is an orthogonal set. Then

(a) the columns of A do not necessarily span $\mathbb{R}^n$.

(b) The matrix A is invertible.

(c) The columns of A are not necessarily independent.

(d) None of the above are true.

(23) Mark each statement True or False.

(a) If $x \in V$ and $\mathcal{B}$ is a basis for V with n vectors, then the $\mathcal{B}$-coordinate vector of x is in $\mathbb{R}^n$.

(b) The vector space $\mathbb{P}_3$ and $\mathbb{R}^3$ are isomorphic.

(c) Let S be a subset of V and every $v \in V$ can be written as a linear combination of the elements in S. Then S is a basis for V.

(d) Let S be a subset of V and every $v \in V$ can be uniquely written as a linear combination of the vectors in S. Then S is a basis for V.

Sample Questions 2-VII

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(24) Let V be a nonzero finite dimensional vector space. Mark each statement True or False.

(a) If there exists a set $\{v_1, \dots, v_p\}$ that spans V, then $\dim V \leq p$.

(b) If there exists a linear independent set $\{v_1, \dots, v_p\}$, then $\dim V \geq p$.

(c) If $\dim V = p$, then there is a spanning set of $p+1$ vectors in V.

(d) If there exists a linearly dependent set $\{v_1, \dots, v_p\}$ in V, then $\dim V \leq p$.

(e) If every set of p elements in V fails to span V, then $\dim V > p$.

(f) If $p \geq 2$ and $\dim V = p$, then every set of $p-1$ nonzero vectors is linearly independent.

(25) Mark each statement True or False.

(a) If $Ax = \lambda x$ for some vector x, then λ is an eigenvalue of A.

(b) If $Ax = \lambda x$ for some value λ, then x is an eigenvector of A.

(c) If v_1 and v_2 are linearly independent eigenvectors, then they correspond to distinct eigenvalues.

(d) An eigenvector of A is in the null space of a certain matrix.

(e) Two similar matrices have the same eigenvalues.

(f) Two similar matrices have the same set of eigenvectors.